Asymptotic calculation of the integral for small β gives

$$t_* = t_e r_i' (T_r/r_i)\,\beta\,(\delta^2/\varphi\varepsilon)^{\delta/\varphi}, \quad t_e = r_i/w(T_r).$$

The induction period t_* shortens more rapidly as T_r increases than does the isothermal evaporation time t_e:

$$t_* \sim t_e^{1+\delta\varphi^{-1}} \sim \exp\left[E\left(1+\delta\varphi^{-1}\right)/RT_r\right].$$

The effective activation energy for the evaporation is $E_* = E\left(1+\delta\varphi^{-1}\right)$ and is dependent on the length of the filament, which governs δ. For a fixed T_r, the lifetime increases rapidly as the filament shortens, which is due to the thermal conduction, which suppresses the growth of temperature inhomogeneities in a short filament.

This calculation has been performed with the Thomson thermoelectric effect neglected. To evaluate the parameter range permitting that approximation, we consider the ratio of the quantities $Q_\lambda = \lambda d^2T/dx^2$ and $Q_s = sjdT/dx$, which govern the power in the dissipative and Thomson sources as appearing in the heat-balance equation (s is the Thomson coefficient, while $j = U/\rho\ell$ is the current density). We take $Q_\lambda \sim \lambda\Delta T/\ell^2$, $Q_s \sim sU\Delta T/\ell^2\rho$ to get $Q_s/Q_\lambda \sim U(s/\rho\lambda)$. Here $s/\rho\lambda$ for conductors does not exceed $5\cdot10^{-5}$ V^{-1}, so for U up to the level of several kV, Q_s/Q_λ is negligibly small.

There is experimental evidence for the lack of effect from the Thomson thermoelectric effect from the coincidence between the critical characteristics (T_*, j_*, U_*, t_*) for direct and alternating currents.

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FLOW PAST A SLIM BODY OF REVOLUTION OF A STATIONARY SUPERSONIC FLOW OF A VIBRATIONALLY EXCITED GAS UNDER A SMALL ANGLE OF ATTACK

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The flow past a body under an angle of attack is of interest within the context of the problem of stability of motion of a body in a gas medium. In ordinary gas dynamics the solution of this problem within the slim body approximation is discussed in [1]. The variability of parameters of flow past the body, generated, for example, by nonequilibrium processes in the gas, may substantially affect the aerodynamic characteristics of the body.

In the present study we consider flow past a slim body of revolution of a vibrationally excited gas at a small angle of attack. The solution obtained makes it possible to calculate the transverse force acting on the body, as well as the torque of this force with respect to the tip of the body. It seems that relaxation of vibrational excitation leads to a change in value, and for a sufficient amount of initial nonequilibrium — even a change of sign of the transverse force. The transverse force also acts on a pointed body (without a rounded slice), while in ordinary gas dynamics the linear theory provides a vanishing transverse force [1].

To investigate this problem the symmetry axis of the body of revolution is conveniently chosen to coincide with the x axis, and the stationary supersonic flow, unperturbed by the

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body, is directed at an angle to this axis. In this case the flow does not possess axial symmetry, and depends substantially on the third cylindrical coordinate – the angle θ. It is assumed that the point $x = 0$ coincides with the tip of the body.

The system of equations, describing the stationary nonaxially symmetric gas flow in cylindrical coordinates, is

$$(\rho u y)_x + (\rho v y)_y + (\rho w)_\theta = 0, \quad M_0^2\rho\left(uu_x + vu_y + \frac{1}{y}wu_\theta\right) + p_x = 0,$$
$$M_0^2\rho\left(uv_x + vv_y + \frac{1}{y}wv_\theta - \frac{w^2}{y}\right) + p_y = 0,$$
$$M_0^2\rho\left(uw_x + vw_y + \frac{1}{y}ww_\theta + \frac{vw}{y}\right) + \frac{1}{y}p_\theta = 0, \tag{1}$$
$$up_x + vp_y + \frac{1}{y}wp_\theta - a^2\left(u\rho_x + v\rho_y + \frac{1}{y}w\rho_\theta\right) = -\rho(\gamma - 1)\left(ue_{kx} + ve_{ky} + \frac{1}{y}we_{k\theta}\right),$$
$$ue_{kx} + ve_{ky} + \frac{1}{y}we_{k\theta} = \omega\left(e_k^* - e_k\right).$$

Here u, v, and w are the gas velocity components along the x, y, and θ axes, respectively, rendered dimensionless at the initial velocity value (at $x = 0$) of the flow unperturbed by the body; ρ, a, and p are the density, frozen speed of sound, and pressure, rendered dimensionless at its initial value (the pressure is still augmented by γ – the adiabatic exponent); e_k and e_k^* are the energy of vibrational degrees of freedom and equilibrium value relative to the square of the initial sound velocity, ω is the reciprocal vibrational relaxation time, rendered dimensionless by the ratio of the initial velocity to the characteristic length of the body L; and $M_0 = u_0/a_0$ (the initial parameter values are denoted by the subscripts 0 in the following). The linear coordinates x and y introduced have been rendered dimensionless (with respect to L).

For e_k^* and ω one can use the equations [2]

$$\omega = k_1 p \exp\left(-k_2 T^{-1/3}\right) L/u_0, \quad e_k^* = \theta_k R/(\exp(\theta_k/T) - 1)/a_0^2, \tag{2}$$

where R is the gas constant, T is the uniform translational temperature, θ_k is a characteristic vibrational temperature, and k_1, k_2 are positive constants depending on the gas properties. The specific k_i values are given in [2].

The problem considered of flow past a slim body of a stationary supersonic flow of a nonequilibrium gas under a small angle of attack α can be separated into two problems within the linear approximation (Fig. 1) – the problem of flow past a slim body with an unperturbed velocity u^a parallel to the symmetry axis of the body (the problem of axial flow past the body), and the problem of flow past a slim body with an unperturbed velocity u^c, directed perpendicular to the symmetry axis of the body (the problem of transverse flow past the body) – similarly to the way this is done in ordinary gas dynamics. In this case it is insignificant that the transverse flow can be subsonic, while the perturbations, carried in the transverse flow by the body, are not small in comparison with the velocity of transverse flow past the body: the solution of this problem is only part of the total solution.

Let u^* be the total flow velocity incident on the body. The following obvious relations hold (Fig. 1)

$$u^a = u^*\cos\alpha, \; u^c = u^*\sin\alpha. \tag{3}$$

In cylindrical coordinates the flow velocity unperturbed by the body (denoted by the superscript 0) is decomposed into the following components along the x, y, and θ axes, respectively:

$$u^0 = u^a, \; v^0 = u^c\cos\theta, \; w^0 = -u^c\sin\theta. \tag{4}$$

The relaxation of vibrationally excited gas molecules leads to energy release, generating stagnation of the supersonic flow. When the flow reaches the speed of sound at some cross section of the flow, it generates the formation of shock waves and breakdown of the stationary flow character. A so-called thermal crisis is evolving, in avoiding which one must be confined to an initial nonequilibrium value corresponding to the dimensionless value. It must be small in comparison with unity, and, thus, in the given problem, in addition to the small parameter δ one succeeds in introducing one more small parameter – the relative initial nonequilibrium

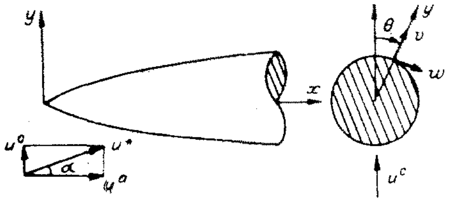

Fig. 1

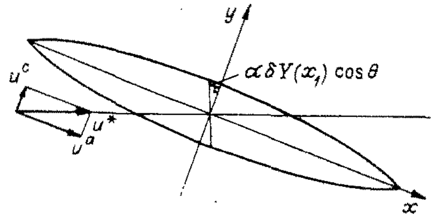

Fig. 2

$$\varepsilon = \frac{e_{k0} - e_{k0}^{*}}{\gamma e_{k0}^{*2} \exp(\theta_k/T_0)(\gamma M_0^2 - 1) + (M_0^2 - 1)/(\gamma - 1)} > 0.$$

We seek a solution of the problem in the form of asymptotic expansions

$$\begin{gathered} u = u_0^a + \varepsilon u_{10} + \delta u_{01} + \ldots, \quad v = (u_0^c + \varepsilon u_{10}^c)\cos\theta + \delta v_{01} + \ldots, \\ w = -(u_0^c + \varepsilon u_{10}^c)\sin\theta + \delta w_{01} + \ldots, \quad p = \frac{1}{\gamma} - \varepsilon p_{10} + \delta p_{01} + \ldots, \\ \rho = 1 - \varepsilon\rho_{10} + \delta\rho_{01} + \ldots, \; e_k = e_{k0} + \varepsilon e_{k10} + \delta e_{k01} + \ldots \end{gathered} \tag{5}$$

For small angles of attack

$$\sin\alpha = \alpha + O(\alpha^3), \quad \cos\alpha = 1 - \frac{\alpha^2}{2} + O(\alpha^4),$$

therefore, from Eqs. (3), (4) we have, accurately to terms of order α^2, inclusively,

$$u_0^a = 1 - \frac{\alpha^2}{2}, \quad u_0^c = \alpha, \quad u_{10}^a = u_{10}, \quad u_{10}^c = u_{10}\alpha.$$

Thus, one more small parameter appears in the analysis – the small angle of attack α. It is assumed that the quantities α, δ, and ε are of the same order of smallness.

We turn now to determine the coefficients in expansions (5). The asymptotic expansion terms containing ε describe the change in gas flow parameters due to energy release during relaxation of the vibrational degrees of freedom to the equilibrium state. We confine ourselves to calculating the principal term in ε. For small angles of attack the difference between the x axis and the flow direction unperturbed by the body is a quantity of order α (Fig. 1). Since this study is confined to calculating the principal terms in expansions in small parameters, it can be assumed that the flow unperturbed by the body is a one-dimensional stationary supersonic flow in the direction of the x axis, implying that $u_{10} = u_{10}(x)$. To find the equations of this flow one must put

$$v^0 = 0, \quad p_y^0 = \rho_y^0 = u_y^0 = e_{ky}^0 = 0.$$

The system obtained from (1) can be transformed to the form

$$\begin{gathered} \rho^0 u^0 = 1, \; p^0 = 1/\gamma + M_0^2(1 - u^0), \; e_k^0 = e_{k0} + M_0^2(1 - u^0)/2 + (1 - a^{02})/(\gamma - 1), \\ u^0 e_{kx}^0 = \omega^0 (e_k^* - e_k)^0. \end{gathered}$$

Substituting expansions (5) into this system, for the coefficients in ε we have the relations

$$u_{10} = \rho_{10} = p_{10}/M_0^2 = (\gamma - 1)e_{k10}/(M_0^2 - 1),$$

where u_{10} is determined by the equation

$$u_{10x} = -\sigma(1 + u_{10}), \quad \sigma = \omega_0\left(\frac{\gamma(\gamma - 1)e_{k0}^{*2}\exp(\theta_k/T_0)(\gamma M_0^2 - 1)}{M_0^2 - 1} + 1\right),$$

with the initial condition $u_{10}(0) = 0$ [following from the condition $u^0(0) = 1$]. Taking into account the initial data, this equation makes it possible to determine

$$u_{10} = \exp(-\sigma x) - 1 \leqslant 0. \tag{6}$$

In calculating the expansion coefficients in δ it is convenient to introduce a function Φ, such that $v_{01} = -\Phi_{xy}$. We then find from system (1)

$$u_{01} = -\Phi_{xx}, \quad p_{01} = M_0^2\Phi_{xx}, \quad w_{01} = -\frac{1}{y}\Phi_{x\theta},$$

$$\rho_{c1} = \Phi_{xx} + \frac{1}{y}(y\Phi_y)_y + \frac{1}{y^2}\Phi_{\theta\theta},$$

$$e_{k01} = \frac{1}{\gamma - 1}\left((1 - M_0^2)\Phi_{xx} + \frac{1}{y}(y\Phi_y)_y + \frac{1}{y^2}\Phi_{\theta\theta}\right),$$

and the function Φ is determined from the equation

$$(M_0^2 - 1)\Phi_{xxx} + (M_0^2 - 1)\sigma\Phi_{xx} - \left(\frac{1}{y}(y\Phi_y)_y + \frac{1}{y^2}\Phi_{\theta\theta}\right)_x -$$
$$- \omega_0\left(1 + \gamma(\gamma - 1)e_{k0}^{*2}\exp\frac{\theta_k}{T_0}\right)\left(\frac{1}{y}(y\Phi_y)_y + \frac{1}{y^2}\Phi_{\theta\theta}\right) = 0, \quad (7)$$

transforming for $\omega_0 \to 0$ to the ordinary equation of small perturbations in the absence of axial symmetry [1]:

$$(M_0^2 - 1)\Phi_{xx} - \frac{1}{y}(y\Phi_y)_y - \frac{1}{y^2}\Phi_{\theta\theta} = 0.$$

As mentioned above, our problem can be considered as a set of problems of axial and transverse flow past the body, therefore the function Φ is represented in the form of a sum of two terms, corresponding to the problems of axial and transverse flows:

$$\Phi(x, y, \theta) = \Phi^a(x, y) + \Phi^c(x, y, \theta).$$

The function Φ^c satisfies condition (7), while for Φ^a the equation is much simpler, since Φ^a is independent of θ:

$$(M_0^2 - 1)[\Phi_x^a + \sigma\Phi^a]_{xx} - \frac{1}{y}(y\Phi_{yx}^a)_y - \omega_0\left(1 + \gamma(\gamma - 1)e_{k0}^{*2}\exp\frac{\theta_k}{T_0}\right)\frac{1}{y}(y\Phi_y^a)_y = 0. \quad (8)$$

We show that the solutions of Eqs. (7) and (8) are interrelated. Differentiating (8) with respect to y and multiplying by $\cos\theta$, one obtains

$$(M_0^2 - 1)[(\cos\theta\Phi_y^a)_x + \sigma\cos\theta\Phi_y^a]_{xx} - (\cos\theta\Phi_y^a)_{xyy} - \frac{1}{y}(\cos\theta\Phi_y^a)_{xy} +$$
$$+ \frac{1}{y^2}(\cos\theta\Phi_y^a)_x - \omega_0\left(1 + \gamma(\gamma - 1)e_{k0}^{*2}\exp\frac{\theta_k}{T_0}\right)\left[(\cos\theta\Phi_y^a)_{yy} + \frac{1}{y}(\cos\theta\Phi_y^a)_y - \frac{1}{y^2}\cos\theta\Phi_y^a\right] = 0.$$

Taking into account that

$$\frac{1}{y^2}(\cos\theta\Phi_y^a)_x = -\frac{1}{y^2}(\cos\theta\Phi_y^a)_{x\theta\theta}, \quad \frac{1}{y^2}\cos\theta\Phi_y^a = -\frac{1}{y^2}(\cos\theta\Phi_y^a)_{\theta\theta},$$

we have

$$(M_0^2 - 1)[(\cos\theta\Phi_y^a)_x + \sigma(\cos\theta\Phi_y^a)]_{xx} - (\cos\theta\Phi_y^a)_{xyy} - \frac{1}{y}(\cos\theta\Phi_y^a)_{xy} -$$
$$- \omega_0\left(1 + \gamma(\gamma - 1)e_{k0}^{*2}\exp\frac{\theta_h}{T_0}\right)\left((\cos\theta\Phi_y^a)_{yy} + \frac{1}{y}(\cos\theta\Phi_y^a)_y\right) -$$
$$- \frac{1}{y^2}(\cos\theta\Phi_y^a)_{x\theta\theta} - \omega_0\left(1 + \gamma(\gamma - 1)e_{k0}^{*2}\exp\frac{\theta_k}{T_0}\right)\frac{1}{y^2}(\cos\theta\Phi_y^a)_{\theta\theta} = 0.$$

Putting now in this equation

$$\Phi^c = \cos\theta\Phi_y^a, \quad (9)$$

one obtains, within the accuracy of Eq. (7), that $\Phi \equiv \Phi^c$. Thus, Φ^c can be determined from Φ^a, using Eq. (9).

In solving Eq. (8), as is usually the case in gas dynamics, it is convenient to transform to the new independent variables ξ and ζ, such that

$$\xi = x - \sqrt{M_0^2 - 1}\,y, \quad \zeta = \sqrt{M_0^2 - 1}\,y.$$

In this case

$$x = \xi + \zeta, \quad y = \zeta/\sqrt{M_0^2 - 1}, \quad \Phi_x^a = \Phi_\xi^a, \quad \Phi_y^a = \sqrt{M_0^2 - 1}(\Phi_\zeta^a - \Phi_\xi^a),$$

$$\Phi_{xx}^a = \Phi_{\xi\xi}^a, \quad \frac{1}{y}(\Phi_y^a y)_y = (M_0^2 - 1)\left(\frac{1}{\zeta}(\Phi_\zeta^a\zeta)_\zeta - \frac{1}{\zeta}(\Phi_\xi^a\zeta)_\zeta - \Phi_{\xi\zeta}^a + \Phi_{\xi\xi}^a\right),$$

$$\Phi_{xxx}^a = \Phi_{\xi\xi\xi}^a, \quad \frac{(\Phi_{xy}^a y)_y}{y} = (M_0^2 - 1)\left(\frac{(\Phi_{\xi\zeta}^a\zeta)_\zeta}{\zeta} - \frac{(\Phi_{\xi\xi}^a\zeta)_\zeta}{\zeta} - \Phi_{\xi\xi\zeta}^a + \Phi_{\xi\xi\xi}^a\right).$$

In the new variables Eq. (8) acquires the form

$$A\Phi^a_{\xi\xi} + (M_0^2 - 1)\left[\Phi^a_{\xi\xi\zeta} + \frac{1}{\zeta}\left((\Phi^a_{\xi\xi} - \Phi^a_{\xi\zeta})\zeta\right)_\zeta\right] + B\left(\frac{(\Phi^a_\zeta\zeta - \Phi^a_\xi\zeta)_\zeta}{\zeta} - \Phi^a_{\xi\zeta}\right) = 0, \tag{10}$$

where

$$A = \omega_0(\gamma - 1)^2\gamma M_0^2 e_{k0}^{*2}\exp\frac{\theta_k}{T_0}; \quad B = -\omega_0\left(1 + \gamma(\gamma - 1)e_{k0}^{*2}\exp\frac{\theta_k}{T_0}\right)(M_0^2 - 1).$$

Equation (10) is linear, while its coefficients are independent of ξ. This equation can be solved by using a Laplace transform in the variable ξ. According to the usual rules of performing this transformation [3] we have

$$\Phi^a(\xi, \zeta) \to \widetilde{\Phi}^a(s, \zeta), \quad \Phi^a_\xi(\xi, \zeta) \to s\widetilde{\Phi}^a(s, \zeta) - \Phi^a(0, \zeta),$$
$$\Phi^a_{\xi\xi}(\xi, \zeta) \to s^2\widetilde{\Phi}^a(s, \zeta) - s\Phi^a(0, \zeta) - \Phi^a_\xi(0, \zeta).$$

We assume vanishing initial conditions:

$$\Phi^a(0, \zeta) = \Phi^a_\xi(0, \zeta) = 0. \tag{11}$$

The condition adopted does not imply that within the approximation considered there is no perturbation in the first characteristic emerging from the tip of the body, since

$$u_{01}(0, \zeta), \quad p_{01}(0, \zeta) \sim \Phi^a_{\xi\xi}(0, \zeta), \quad v_{01}(0, \zeta) \sim \Phi^a_{\xi\zeta}(0, \zeta) - \Phi^a_{\xi\xi}(0, \zeta).$$

For $\tilde{\Phi}^a$ we obtain the ordinary differential equation

$$\widetilde{\Phi}^a_{\zeta\zeta} + \left(\frac{1}{\zeta} - 2s\right)\widetilde{\Phi}^a_\zeta + s\left(-\frac{1}{\zeta} + \frac{As}{B - (M_0^2 - 1)s}\right)\widetilde{\Phi}^a = 0, \tag{12}$$

which is usually solved as follows. Instead of $\tilde{\Phi}^a$ we introduce the new independent function $z(s, \zeta)$:

$$\widetilde{\Phi}^a(s, \zeta) = z(s, \zeta)\exp(s\zeta).$$

Instead of (12) we then have the following equation for z

$$z_{\zeta\zeta} + \frac{1}{\zeta}z_\zeta - s^2\left(1 - \frac{A}{B - (M_0^2 - 1)s}\right)z = 0,$$

which by the replacement

$$z = \frac{\hat{z}}{s\sqrt{1 - \frac{A}{B - (M_0^2 - 1)s}}}, \quad \zeta = \frac{\hat{\zeta}}{s\sqrt{1 - \frac{A}{B - (M_0^2 - 1)s}}}$$

is reduced to the Bessel equation of zeroth order

$$\hat{z}_{\hat{\zeta}\hat{\zeta}} + \frac{1}{\hat{\zeta}}\hat{z}_{\hat{\zeta}} - \hat{z} = 0. \tag{13}$$

The solution of Eq. (13) is well known [4], and is expressed in general form in terms of the modified Bessel functions of zeroth order K_0, I_0:

$$\hat{z} = C_1(s)K_0(\hat{\zeta}) + C_2(s)I_0(\hat{\zeta}).$$

The solution of interest to us must be restricted to $\hat{\zeta} \to \infty$, and consequently one must put $C_2(s) = 0$. Returning to the variable ζ of the function $\tilde{\Phi}^a$, we find

$$\widetilde{\Phi}^a = C_1(s)\frac{K_0\left(s\sqrt{1 - \frac{A}{B - (M_0^2 - 1)s}}\zeta\right)}{s\sqrt{1 - \frac{A}{B - (M_0^2 - 1)s}}}\exp(s\zeta).$$

Since our purpose is to determine the forces acting on the slim body, we consider the behavior of the solution for small $\hat{\zeta}$. It is well known [4] that

$$K_0(\eta) \sim -\ln(\eta/2), \; I_0(\eta) \sim 1, \; \eta \to 0. \tag{14}$$

Assuming that s is sufficiently large, we expand the expression under the sign of the square root in powers of $1/s$:

$$\sqrt{1-\frac{A}{B-(M_0^2-1)s}}=1+\frac{A}{2(M_0^2-1)}\frac{1}{s}+O\left(\frac{1}{s^2}\right)\approx 1+\frac{\Lambda}{s},\ \Lambda=\frac{A}{2(M_0^2-1)}.$$

We take into account the asymptote (14), as well as the representation of the exponential function by the series $\exp(s\zeta) = 1 + s\zeta + \ldots$, and for $\zeta \to 0$ we obtain

$$\tilde{\Phi}^a=-C_1(s)\frac{\ln\left(\frac{s+\Lambda}{2}\zeta\right)}{s+\Lambda}.$$

We turn now to inverting the Laplace transform of the function $\tilde{\Phi}^a$. The original function for the expression $\frac{-1}{s+\Lambda}\ln\left(\frac{s+\Lambda}{2}\zeta\right)$ is a combination of the functions [3] $\exp(-\Lambda\xi)\ln\frac{2\xi}{\zeta}$. Let C_1 have the original function $g(\xi)$ [in gas dynamics $g(\xi)$ is the so-called "source intensity function"], then by the rule of the representation product [3] we find

$$\Phi^a=\int_0^\xi g(\eta)\exp(-\Lambda(\xi-\eta))\ln\frac{2(\xi-\eta)}{\zeta}\,d\eta.$$

Selecting the principal term in y (for $y \to 0$) and using (9), we find

$$\Phi^c=-\cos\theta\frac{1}{y}\int_0^x g(\eta)\exp(-\Lambda(x-\eta))\,d\eta.$$

To determine g one must consider the boundary conditions on the surface of the slim body. It is the nonleaking condition through the surface of the body and, if the generatrix of the body is $Y = Y(x)$, it acquires the form

$$\delta Y_x=\frac{v}{u}. \tag{15}$$

In any plane transverse to the secant of the body the transverse velocity is

$$v=(u_0^c+\varepsilon u_{10}^c)\cos\theta+\delta v_{01}=\alpha(1+\varepsilon u_{10})\cos\theta-\delta\Phi_{xy}.$$

The axial velocity is

$$u=u_0^a+\varepsilon u_{10}=1+\varepsilon u_{10},$$

and equality (15) for the principal expansion terms is rewritten as

$$\delta Y_x=\alpha\cos\theta-\delta\Phi_{xy}^a-\delta\Phi_{xy}^c.$$

The expressions obtained can be divided into two parts, corresponding to axial and transverse flows:

$$Y_x=-\Phi_{xy}^a,\quad 0=\alpha\cos\theta-\delta\Phi_{xy}^c. \tag{16}$$

The first equality in (16) corresponds to the boundary condition for axial flow past the slim body, and the second makes it possible to determine the function g.

Replacing the variable of integration $t = x - \eta$ in the expression for Φ^c and differentiating it with respect to x, we obtain

$$\Phi_x^c=-\cos\theta\frac{1}{y}\left(g(0)\exp(-\Lambda x)+\int_0^x g'(x-t)\exp(-\Lambda t)\,dt\right). \tag{17}$$

Returning to the variable $\eta = x - t$, following differentiation of (17) with respect to y and using the second equality of (16) we have

$$\alpha\delta Y^2\exp(\Lambda x)=g(0)+\int_0^x g'(\eta)\exp(\Lambda\eta)\,d\eta,$$

whence it is seen that $g(0) = 0$ and $Y(0) = 0$ (the body contour is closed at the tip of the body). Now $g_1'(x)$ acquires the form

$$g_1'(x)=\alpha\delta\exp(-\Lambda x)(Y^2\exp(\Lambda x))_x. \tag{18}$$

Substituting (18) into (17), we find

$$\Phi_x^c = -\alpha\delta\frac{\cos\theta}{y}\int_0^x (Y^2(\eta)\exp(\Lambda(\eta - x)))_\eta\, d\eta = -\alpha\delta\frac{\cos\theta}{\pi y} S(x) \tag{19}$$

($S = \pi Y^2$ is the area of the transverse cross section of the body).

We turn now to determining the transverse force. The projection of the surface area element of the body, being orthogonal to the radius of the body, is $L\delta Y d\theta dx$. The radial component of the force acting on this area element equals

$$(p - p_0)^c L\delta Y d\theta dx. \tag{20}$$

To obtain now the transverse force (directed parallel to the velocity u^c of the transverse flow unperturbed by the body), it is necessary to multiply (20) by $-\cos\theta$ and integrate over the whole surface of the body:

$$N = -\int_0^1\int_0^{2\pi} (p - p_0)^c \cos\theta\delta Y L^2\, d\theta\, dx.$$

We express the pressure at the subrace of the slim body $p - p_0$ in terms of the velocity components of the gas flow past the body. For this we use the integral equation of motion along the stream lines (the Bernoulli integral)

$$\frac{1}{2}\gamma M_0^2(u^2 + v^2 + w^2) + \int_1^{\gamma p}\frac{1}{\rho}\, d\widetilde{p} = \frac{1}{2}\gamma M_0^2. \tag{21}$$

According to (3)-(5) we have

$$u^2 + v^2 + w^2 = (1 + \varepsilon u_{10} + \delta u_{01})^2 - \alpha^2(1 + \varepsilon u_{10})^2 + ((1 + \varepsilon u_{10})\alpha\cos\theta + \delta v_{01})^2 + (-(1 + \varepsilon u_{10})\alpha\sin\theta + \delta w_{01})^2.$$

The boundary condition (15) gives

$$v = (1 + \varepsilon u_{10})\alpha\cos\theta + \delta v_{01} = u\delta Y_x = (1 + \varepsilon u_{10})\delta Y_x.$$

The term corresponding to w is transformed by using (19), since

$$\delta w_{01} = -\frac{\delta}{y}\Phi_{x\theta}^c.$$

We find

$$(1 + \varepsilon u_{10})\alpha\sin\theta - \delta w_{01} = (1 + \varepsilon u_{10})\alpha\sin\theta + \frac{\delta}{y}\Phi_{x\theta}^c = 2(1 + \varepsilon u_{10})\alpha\sin\theta.$$

Thus,

$$u^2 + v^2 + w^2 = (1 + \varepsilon u_{10} + \delta u_{01})^2 + (1 + \varepsilon u_{10})^2((\delta Y_x)^2 + (4\sin^2\theta - 1)\alpha^2),$$

and by (21) we obtain

$$\frac{1}{2}M_0^2(2\delta u_{01} + (\delta Y_x)^2 + (4\sin^2\theta - 1)\alpha^2) + \delta p_{01} = 0.$$

As a result

$$\delta p_{01} = M_0^2\left[\Phi_{xx}^a + \delta\Phi_{xx}^c - \frac{1}{2}(\delta Y_x)^2 - \frac{1}{2}(4\sin^2\theta - 1)\alpha^2\right].$$

This expression can be divided into two parts, corresponding to the axial and transverse flows:

$$\delta p_{01}^a = M_0^2\left[\delta\Phi_{xx}^a - \frac{1}{2}(\delta Y_x)^2\right],$$

$$\delta p_{01}^c = M_0^2\left[\delta\Phi_{xx}^c - \frac{1}{2}(4\sin^2\theta - 1)\alpha^2\right].$$

We differentiate (19) with respect to x, and for $y = \delta Y(x)$ we have

$$\Phi_{xx}^c = -\frac{\alpha}{\pi y}\cos\theta\delta S'(x) = -2\alpha\cos\theta Y_x.$$

Since the flow unperturbed by the body takes place at an angle to the body axis, relaxation of the vibrationally excited gas molecules in the unperturbed flow also leads to generation of a pressure gradient in a direction perpendicular to the body axis. A transverse force is consequently developed, acting on the body. The flow unperturbed by the body is a plane parallel flow. Let, on the surface of the body and at the point $x = x_1$, $y = 0$, $\theta = \pi/2$ (Fig. 2),

$$p = \frac{1}{\gamma} - \varepsilon p_{10}(x_1).$$

Then, at each point of the surface of the slim body $y = \delta Y(x_1)\cos\theta$ the pressure at the cross section $x = x_1$ is determined by the relation

$$(p = \frac{1}{\gamma} - \varepsilon p_{10}(x_1 + \alpha\delta Y(x_1)\cos\theta) = \frac{1}{\gamma} - \varepsilon p_{10}(x_1) - \varepsilon\alpha\delta p_{10x}(x_1)\, Y(x_1)\cos\theta.$$

The pressure variation in the direction transverse to the body axis and along the surface of the body is, thus, the quantity

$$-\varepsilon\alpha\delta p_{10x}(x_1)\, Y(x_1)(\cos\theta + 1) = \varepsilon\alpha\delta M_0^2\sigma\exp(-\sigma x_1)\, Y(x_1)(\cos\theta + 1) > 0.$$

This is all necessary to calculate the variation in pressure along the surface of the body

$$(p - p_0)^c = \gamma p_0(-\varepsilon p_{10}^c + \delta p_{01}^c) = \gamma p_0(\varepsilon\alpha\delta M_0^2\sigma\exp(-\sigma x)Y(x)(\cos\theta + 1) + \delta p_{01}^c),$$

from which we obtain the following expression for the transverse force

$$\overline{N} = N/(\rho_0 u_0^2 L^2) = \alpha\delta\int_0^1\int_0^\pi \cos\theta Y(x)\left[4\cos\theta Y_x + \alpha(4\sin^2\theta - 1) - 2\varepsilon\delta M_0^2\sigma\exp(-\sigma x)\, Y(x)(\cos\theta + 1)\right] dx\, d\theta.$$

Note that

$$\int_0^\pi \cos^2\theta\, d\theta = \frac{\pi}{2}, \quad \int_0^\pi \cos\theta(4\sin^2\theta - 1)\, d\theta = 0$$

and the term of order $\alpha\delta^2$ falls off, therefore

$$\overline{N} = \delta^2\alpha\int_0^1 S'(x)\, dx - \varepsilon\delta^2\alpha\int_0^1 M_0^2\sigma\exp(-\sigma x)\, S(x)\, dx.$$

Carrying out the integration in the first term, we finally find

$$\overline{N} = \delta^2\alpha S(1) - \varepsilon\delta^2\alpha\int_0^1 M_0^2\sigma\exp(-\sigma x)\, S(x)\, dx. \tag{22}$$

Here S(1) is the area of the rounded slice of the body. For complete determination of the transverse force it is still necessary to find the contribution to the transverse force resulting from the given pressure [1]. If the body is pointed, i.e., $S(1) = 0$, instead of (22) we write down

$$\overline{N} = -\varepsilon\delta^2\alpha\int_0^1 M_0^2\sigma\exp(-\sigma x)\, S(x)\, dx < 0. \tag{23}$$

The calculation of terms of order $\varepsilon\delta$ and δ^2 in the parameter expansions is not provided due to their unwieldiness. The contributions to the transverse force due to these terms are of the orders $\varepsilon\delta^3\alpha$ and $\delta^4\alpha$. In ordinary gas dynamics the transverse force, including second-order approximate terms, was given in [5].

We determine the torque of the transverse force with respect to the tip of the body $x = 0$ in the form

$$\overline{M} = \delta^2\alpha\int_0^1 xS'(x)\, dx - \varepsilon\delta^2\alpha\int_0^1 xM_0^2\sigma\exp(-\sigma x)\, S(x)\, dx.$$

The first term is integrated by parts

$$\int_0^1 xS'(x)\, dx = S(1) - \int_0^1 S(x)\, dx.$$

For a pointed body

$$\overline{M} = -\delta^2\alpha\int_0^1 S(x)\, dx - \varepsilon\delta^2\alpha\int_0^1 xM_0^2\sigma\exp(-\sigma x)\, S(x)\, dx.$$

TABLE 1

T_k, K	σ	Λ	ω	ε	N_1	N_2	N
500	2,321	0,0041	2,297	0,0006	0,2545	—0,019	0,235
1000				0,0258		—0,861	—0,606
2000				0,1672		—5,587	—5,332
3000				0,3547		—11,86	—11,6

TABLE 2

T_k, K	σ	Λ	ω	ε	N_1	N_2	N
500	3,319	0,0002	3,318	0,0025	0,2545	—0,079	0,175
1000				0,0565		—1,809	—1,555
2000				0,3201		—10,25	—9,994
2500				0,4831		—15,47	—15,21

We calculate N for a body of revolution, given by the information available in [2] concerning the flow parameters and the constants characterizing the physical properties of the gas. Let the generatrix of the body be given by the equation $Y=x(1-x)$ $(0\leqslant x\leqslant 0.9)$. We put $\delta = 0.1$, $\alpha = 0.1$. The calculation results for molecular nitrogen and carbon monoxide are given in Tables 1 and 2, where N_1 is the transverse force for the case of ordinary gas dynamics $\delta^2\alpha S(1)$, N_2 is calculated by Eq. (23), and N – by Eq. (22) (the given transverse force values must be multiplied by 10^{-4}). In the calculations it was assumed that $M_0 = 2$, $p = 10^5$ Pa, T = 430 K (for molecular nitrogen), T = 250 K (for carbon monoxide).

Thus, unlike ordinary gas dynamics [1] a pointed slim body in a flow of a vibrationally excited gas experiences a transverse force, directed toward the transverse flow and tending to increase the angle of attack α.

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